Competitive Resource Allocation based on Priority

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Abstract: According to basic characteristics of resource allocation problem based on priority: competitive resource, the concepts of consistency and difference of preference is proposed in this paper. The limited quantity of resources and consistency of the preference is the cause of resource competition. Under condition of consistency of preference in perfect information environment, the outcome of resource allocation depends on structure of priority, and the consistency of priority structure is necessary condition to ensure fairness of resource allocation.

Keywords: Priority; Consistency; Indivisible Goods ; Acyclicity

I. Introduction

The problem of resource allocation based on priority was common in public administration field, such as college and middle school enrollment, offices and facilities allocation in institutions. In this field, the main research direction is seeking the excellent allocation mechanism by comparison. Here are main research results: Balinski and Sönmez[4] confirmed the efficiency superiority of deferred accepted algorithm, but they also showed that efficiency is incompatible with fairness in any mechanism; Ergin[12] found that the limitation of priority structure could guarantee the performance of GS mechanism: only in the condition of Ergin acyclicity, the AOSM (Agent-optimal stable mechanism) is strategy-proof, Pareto efficiency and consistency. Usually, priority structure is exogenous, which is included in resource allocation problems. Ergin' research inspired a new analysis about how to find the corresponding allocation mechanism on the base of different priority structures.

At present, most researches overemphasize the priority structure, but neglect the preference structure. Preference structure is a set of preference order of participants for allocated resource. In a lot of competitive resource allocation, the preference of participants is consistency, such as school choice problem, students have the same order list of the quality of school; the consistency of preference also influence the performance of mechanisms. For example, in the case of Ergin's subversion of priority structure, if the preference of students is consistent, it could be *abc*, *acb*, *bac*, *bca*, *cab*, *cba*, while the interesting thing is that under AOSM, TTC and BOS mechanisms, the result is the same. The subversion of priority structure is superimposed effect of special priority structure and special preference structure. It means that if all students have the identical preference, under resource allocation with cyclic priority structure, the results of TTC and AOSM mechanisms will unlikely be contrary to fairness principle. Meanwhile, the definition of acyclic priority structure by Ergin is not exact. The effect of priority structure is to coordinate the competitive demands for resources. In the competitive environment, cycle structure must effect the allocation of resource. Hence, this paper will use consistent priority structure instead of acyclic priority structure.

This paper will analyze the preference structure, and make a definition of consistent preference structure and discrepant preference structure. To aim at the resource allocation problems with consistent preference structure, there leads to a new definition about resource competition. Actually, the key point of competitive allocation is priority. While under extensive preference condition, we defined non-competitive resource allocation. Considering the priority structure, according to Ergin' research result [12][15][17]：Only under the consistent priority structure, the performance of mechanisms could be efficient, fair and consistent, which we would like to focus on as follows: first, under consistent preference structure and cyclic priority structure, could the outcome of mechanism be efficient, fair and consistent? Second, in double consistency environment, the problem of allocation is concise; could we low down the demands of mechanisms?

II. The resource allocation problem based on priority

Model

Let $N=\{1,2,...,n\}$ be a set of a limited number of participants, X is a set of limited unattached resource. Suppose x , y as elements of the set X . If the participant is not assigned to any element in the set X , then it is assigned to null object, which is denoted by ϕ . Let $q_x \ge 1$ be the available number x could use or the supply x could give, suppose $q \equiv (q_x)_{x\in X}$ as supply vector of resource. Every participant $i \in N$ have complete, transferable and strict preference R_i for resource $X \cup \{\phi\}$, let $\Re$ be the set of all preferences R_i , and the correspond strict preference relationship denoted by P_i. Let S^N be $N \to N$ un-directional sequence set, as for $x \in X$, $f_x \in S^N$ is priority ranking for participant allocating resource x, participant i is denoted by $f_x(i)$ in f_x, $f_x(i) < f_x(j)$ means i has a higher priority to j. The set of priority rankings of all resources is called priority structure, denoted by $f \equiv (f_x)_{x \in X}$. Usually, suppose priority structure is exogenous. $N_x \subseteq U_x^f(i)$ denotes the set of participants who have higher priority to i in the process of allocating resource x.

Let preference domain $R \equiv (R_i)_{i \in N} \in \Re^N$ be the allocation problems based on priority. $\alpha \equiv (\alpha_i)_{i \in N}$ denotes a outcome set with two satisfied conditions: (1) every participant could get object no more than one, (2) the object number assigned to participant could not exceed the supply number. Null object could be assigned to participant without any number limited. Let A be the set of all allocations. Mechanism φ is a function with problem $R \in \Re^N$ to outcome $\varphi(R) \in A$. Given $R \in \Re^N$, suppose $\varphi_i(R)$ as participant i 's outcome in total outcome $\varphi(R)$. Suppose $M \subseteq N$, and $R \in \Re^N$。Let R_M be preference domain $(R_i)_{i \in M}$, and $\varphi(R)|_M$ be $\varphi_i(R)_{i \in M}$. While let s_{-x} be $(s_y)_{y \in X \setminus \{x\}}$.

Allocation principles

We need some index to evaluate the performance of resource allocation, and next is an introduction of some index reminded in literature.

Usually, Pareto-efficiency is supposed as the efficiency index, an efficient allocation is the one which want to improve someone's welfare, and have to sacrifice some others'. As for allocation problem $R \in \Re^N$, if there doesn't exist the outcome $\alpha \in A$, making: as for some participants $i \in N$, there is $\alpha_i R_i \varphi_i(R)$, while as for some other participants $j \in N$, there is $\alpha_j P_j \varphi_j(R)$, then φ is efficient mechanism, and $\varphi(R)$ is Pareto-efficient.

Balinski and Sönmez[4] leads fairness into allocation problems based on priority: in a fair allocation, no participant envy the other participant who has a lower priority. The definition of fairness is: as for all $i, j \in N$ and all $R \in \Re^N$, $\varphi_j(R) P_i \varphi_i(R) \Rightarrow f_{\varphi_j(R)}(j) < f_{\varphi_j(R)}(i)$. According to Balinski and Sönmez' research [4], no mechanism could satisfy these two appealing properties: efficiency and fairness.

The change of resource (increasing or decreasing) will influence participants in the same way, this is consistency. Let X be the set of initial resource, s be initial supply vector, and the problem is $(s'_x, s_{-x}, R), s'_x \le s_x$, outcome is $\varphi(s'_x, s_{-x}, R)$, the definition of resource monotonicity [6][17][23] is: as for all $x \in X$, $s'_x \le s_x$, $R \in \Re^N$, there has: as for all $i \in N$, $\varphi_i(s, R) R_i \varphi_i(s'_x, s_{-x}, R)$ or as for all $i \in N$, $\varphi_i(s'_x, s_{-x}, R) R_i \varphi_i(s, R)$. Population monotonicity[17][22]: let X be initial object set, s is initial supply vector, N is initial participant set, and the problem is $(N', (R_i)_{i \in N'}), N' \subseteq N$, outcome is $\varphi(N', (R_i)_{i \in N'})$, the definition of population monotonicity is: as for all $N' \subseteq N$ and all $R \in \Re^N$, there is: as for all $i \in N'$, with $\varphi_i(N, R) R_i \varphi_i(N', (R_i)_{i \in N'})$ or as for all $i \in N'$, with $\varphi_i(N', (R_i)_{i \in N'}) R_i \varphi_i(N, R)$. Put these two index together, we could get consistency index [10][12] [17]: let N be initial participant set, X be initial object set, s be initial supply vector, and the problem is $(N', s', (R_i)_{i \in N'}), and\ N' \subseteq N, s' \le s$, the outcome is $\varphi(N', s', (R_i)_{i \in N'})$, let $r_{N'}^{\varphi}(N, s, R)$ be reduced problem which is produced by original problem (N, s, R) after the leaving of participant $N \setminus N'$ with the outcome of φ, thus the definition of consistency is: given $N' \subseteq N$ and $R \in \Re^N$, as for $x \in X$, $s'_x \equiv s_x - \left|\{ j \in N \setminus N' : \varphi_j(N, s, R) = x \}\right|$, $\phi \ne N' \subset N$ and $R \in \Re^N$, then $\varphi(N, s, R)|_{N'} = \varphi(r_{N'}^{\varphi}(N, s, R))$.

III. Priority structure

The priority structure in literature is mainly about acyclic priority structure, and the corresponding definitions include: Ergin acyclicity [12], Kesten acyclicity[17], Weak X-acyclicity and strong X-acyclicity [15]. According to their definitions, acyclicity is the joint character of priority structure and fixed amount, thus acyclicity condition includes: (1)cyclic priority structure doesn't exist; (2)even if the cycle exist, there is enough fixed amount to prevent the cycle's function.

At first, we would like to analyze the priority without cycle. Let $N = \{1, 2, ..., n\}$ be the set of all participants, then the priority of attached goods $x \in X$ is $f_x = \{ f_x(1), ..., f_x(n) \}$, priority structure is $f \equiv (f_x)_{x \in X}$. In these definitions, X cycle is the minimum unit, the other cycles include at least one X cycle.

Definition 1: if there is no cycle structure in priority structures of different goods, as for any $i, i' \in N$, and any $x, x' \in X$, no $f_x(i) < f_x(i')$ and $f_{x'}(i') < f_{x'}(i)$ exist, then priority structure $f \equiv (f_x)_{x \in X}$ is consistent priority structure.

In the consistent priority structure, the priority ranking of all the goods is the same or at least the frontal ranking is the same. For an example, here are goods a、b、c, participants 1、2、3、4, the priority ranking of goods a is (4,3,2,1), and b is (4,3,2), c is (4,3,2,1). The ranking of a and c is the same, the priority ranking of b, a and c is consistent. In resource allocation problems, under the consistent priority structure, top ranking participants always have a better priority in every resource allocation. Such as college selecting, the student who get the top score always has the top priority in any college.

Secondly, according to Ergin's research, scarcity condition means enough participants who get the priority lead to competition for attached resources. In the definition of acyclicity, the meaning of fixed amount vector is not exact. On one side, priority structure itself could coordinate competition. If this competition does not exist, priority structure will lose its significance. Hence, in the resource allocation based on priority, the hypothesis that cycle condition doesn't work is false. In another word, set up inconsistent priority, there must be cyclic priority structures. On the other side, if consistent priority exists, the competition based on priority and fixed amount will lost its significance. Officially, here we have:

Proposition 1: in the resource allocation problems based on priority, inconsistent priority structure must lead to cyclic priority structure.

Therefore, we conclude that acyclic priority is meaningless. We will use consistent priority structure and cycle priority structure instead of cyclic and acyclic priority structure.

IV. Preference structure

Preference structure is the relationship between different participants' preferences. In a lot of resource allocation problems, participants show the same preference rankings for resource, we call it the consistency of preference structure. Officially, the consistency of preference could define as:

Definition 2: as for any $i,j\in N$, and any $x,x'\in X$, if there don't exist xP_ix', $x'P_jx$, then the preference structure of participants is consistent.

In the consistent priority structure, the preference ranking of all participants is the same or at least the frontal ranking is the same. For an example, here are goods a、b、c, participants 1、2、3, the preference ranking of participant 1 is (a, b, c), and 2 is (a, b), 3 is (a, b, c). The preference ranking of 1 and 3 is the same; the priority ranking of 2, 1 and 3 is consistent.

If the amount of resource is less than the amount of applicants (every applicant only could get one unit resource), this leads to the scarcity of resource. According to Ergin's definition: given priority structure f, as for any $x,x'\in X$, and any $i,j,l\in N$, exist(may be null) disjoint set $N_x,N_{x'}\in N\setminus\{i,j,l\}$, make $N_x\subseteq U_x^f(j),N_{x'}\subseteq U_{x'}^f(i)$, while $|N_x|=q_x-1$ and $|N_{x'}|=q_{x'}-1$. Obviously, the scarcity of resource depends on every kind of supply q_x and demand $N_x\subseteq U_x^f(j),N_{x'}\subseteq U_{x'}^f(i)$. When the demand exceeds supply, the competition emerges. Ergin's definition applied to every kind of resource which has special priority, if it is in the consistent priority structure, $N_x\subseteq U_x^f(j),N_{x'}\subseteq U_{x'}^f(i)$ could not describe the demand, $|N_x|=q_x-1$ and $|N_{x'}|=q_{x'}-1$ also could not describe the scarcity and competition of resource.

Example 1: here are a，b，c three kinds of resource, there are one units of a, and two each of b and c. there are six applicants 1，2，3，4，5，6. the priority structure and preference structure are as follows:

PRIORITY STRUCTURE							PREFERENCE STRUCTURE	1	2	3	4	5	6
a	6	5	4					a	a	a	a	a	a
b	6	5	4	3	2			b	b	b	b	b	
c	6	5	4	3	2	1		c	c	c			

In this example, the preference structure and priority structure are both consistent. At first, the amount of resource denotes limitation of supply. Secondly, demand of resource could be reflected in two sides: preference structure and priority structure. Here are 6 participants demand for resource a, 5 for b, and 3 for c. however, there are 6 participants have priority to c, 5 to b, and 3 to a. we could see that, under the structure of consistent priority, preference could display the competition better, it is false to describe competition based on priority.

Proposition 2: as for the allocation problem with consistent preference structure, higher ranking is, stronger competition is.

As for $i,j\in N$, and $x,x'\in X$, if there exist xP_ix' and xP_jx', while the amount of x is limited, this means these two are competitor for x.

Definition 3: if there exist both preference consistency and scarcity ($N>q_x$), we call it competitive matching market.

In competitive matching market, the game between participants is non-cooperative game. When trade cycle does not exist, on the condition of no monetary payoff, there is no trade exist. If we use $U_i(x)$ denotes the utility when participant i obtain resource x, the cardinal utility of preference consistency could describe as for any $i,j\in N$, and $x,x'\in X$, if $U_i(x)>U_i(x')$, then $U_j(x)>U_j(x')$. There may have this kind of cases in expression of cardinal utility: there is a participant, who prefer x to x', it means that $U_i(x)-U_i(x')>U_j(x)-U_j(x')$, it could improve these two's welfares $[U_i(x)-U_i(x')]>m>[U_j(x)-U_j(x')]$ by monetary payoff. The key point of mechanism design in competitive matching markets is to corporate interest conflict between participants by the design of priority structure, such as students' scores in school choice.

If there is preference difference in matching mechanism, under the limitation of resource scarcity, there could form trade cycle.

Definition 3: as for any $i,j\in N$, and $x,x'\in X$, if there exist xP_ix' and $x'P_jx$, then there is difference between the preference of i,j

Example 2: $i,j,k,l\in N$, they choose four schools $x_a,x_b,x_c,x_c\in X$, and each school has one position，preference relationship of the four students are as follows:

$i \quad x_a \succ x_b \succ x_c \succ x_d$

$j \quad x_d \succ x_b \succ x_c \succ x_a$

$k \quad x_c \succ x_d \succ x_a \succ x_b$

$l \quad x_b \succ x_c \succ x_d \succ x_a$

There is difference between the relationships above: each student could get their preferred resource by the design of allocation mechanism $\begin{Bmatrix} i & j & k & l \\ x_a & x_d & x_c & x_b \end{Bmatrix}$. This difference guarantees cooperative relationship among participants. Especially, all participants could get their first choice, and no competition exists. We call this case cooperative resource allocation problem.

Definition 4: When the resource could satisfy all participants their first choices, it is cooperative resource allocation problem.

Proposition 3: cooperative resource allocation problem is no need to set up priority.

V. Consistent preference structure and allocation performance

According to Ergin's research: In any allocation problem (f,q) reached by AOSM, the following expression is equivalent: (i) $\varphi_f(R)$ is Pareto-efficient; (ii) $\varphi_f(R)$ strategy-proof; (iii) $\varphi_f(R)$ is consistent. (iv) f is acyclic.

Acyclic priority structure is necessary and sufficient condition of Pareto-efficiency, strategy-proofness and consistency of outcome. This paper will confirm that: under the consistent preference structure, (1) even if the priority structure is cycle, the outcome only depends on priority structure, and it is Pareto-efficient, strategy-proof and consistent. (2) if priority structure is acyclic, the outcome has nothing to do with the mechanism.

Under cycle priority structure

Example 3: here are indivisible goods a, b, c，the amount of each is 1，the preference is consistent preference structure, priority is cycle structure, their relationships are as follow:

PRIORITY STRUCTURE					PREFERENCE STRUCTURE	i	j	k	l
a	i	j	k	l		c	c	c	c
b	j	k	l	i		b	b	b	b
c	k	l	i	j		a		a	

Under Agent-optimal stable mechanism[17], TTC mechanism and BOS mechanism, all outcome is $\begin{Bmatrix} a & b & c \\ i & j & k \end{Bmatrix}$.

At first, under AOSM mechanism, round 1，all participants submit application c，k is listed as c's candidate and reject other applicants; round 2, i、j、l submit application b，j is listed as candidate and reject i、l; round 3, i、l apply to a，i is listed as a candidate and reject l; the procedure terminates when all the resource have candidates. Then we could get this outcome. Under TTC mechanism，because of non-formation of trade cycle, the allocation procedure is the same with AOSM, the outcome, either. Under BOS mechanism, all participants submit their application according to their highest priority; we can have the same outcome.

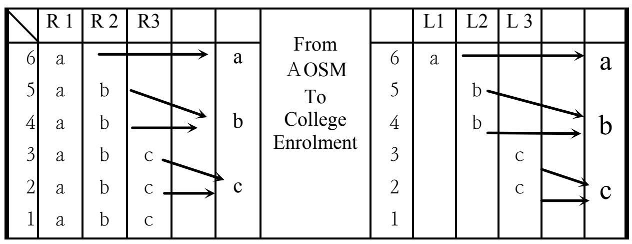
Except the same outcome of different mechanism, the outcome completely depends on priority, furthermore, depends on the highest ranking of priority structure

Theorem 1: under the preference consistency, the outcome of TTC、AOSM and BOS mechanism depend on priority.

Theorem 2: under the preference consistency, the outcome of TTC and AOSM mechanism is Pareto-efficient、fair and consistent, BOS mechanism is Pareto-efficient and consistent.

Under consistent priority structure

Example 1 showed consistent priority structure. Obviously, under AOSM、TTC、BOS and college enrolment mechanism, all the outcome is $\begin{Bmatrix} a & b & c \\ 6 & 4,5 & 2,3 \end{Bmatrix}$，the following tables are to describe different mechanism.



From the solving process of example 1, we could see that, under different mechanism, with the same consistent preference and priority structure, the outcome is the same, do it imply that under certain assumption, the outcome of allocation has nothing to do with mechanism? The following theories will confirm that the answer is positive.

Theorem 3: under the condition of consistent preference and priority structure, the outcome of allocation has nothing to do with mechanism.

VI. Summary

The problem of resource allocation based on priority was widely applied to public administration fields and ordinary institutions，many researchers did a lot constructive work in this field, Balinski and Sönmez[4] confirmed the efficiency superiority of deferred accepted algorithm, but there is no mechanism could satisfy these two appealing properties(efficiency and fairness) at the same time; Ergin[12] found that only by the limitation of priority structure, it could guarantee the efficiency of GS mechanism: only in the condition of Ergin acyclicity, the ASOOM mechanism is strategy-proof, Pareto efficiency and consistency. However, we know that, the performance of allocation mechanism is also influenced by preference structure of participants, such as example 2, although the priority structure is cycle, under consistent preference structure, TTC and ASOM mechanism both could bring out a good performance with efficiency, fairness and consistency. Fewer scholar pay attention on the function of preference structure in mechanisms, in addition, the cyclic condition listed in previous literature could not reflect actual situation, based on these considerations, this paper defines priority consistency and preference consistency, furthermore, we use consistent priority instead of (a)cyclic priority, and make a research on the influence of consistent preference structure on the performance and outcome, in the condition of cycle priority and consistent priority. We have a conclusion: first, under the preference consistency, the outcome of TTC, AOSM and BOS mechanism is Pareto-efficient, fair and consistent; second, under the condition of consistent preference and priority structure, the outcome of allocation has nothing to do with mechanism.

Appendix

Proof of theorem 1

Every allocation problem could be denoted by a matrix, and so does this theorem. Rows of matrix represent different participants, and columns represent different kinds of resource, the element of the i th row and the j th column is composed of two numbers, the left one indicates resource j 's ranking in participant i 's preference list. If j is unacceptable to i , then the number be ∞ , while the right one indicates participant i 's ranking in resource j 's priority list. Here is example 2 which is denoted by a matrix:

	a	b	c
i	3,1	2,4	1,3
j	∞,2	2,1	1,4
k	3,3	2,2	1,1
l	∞,4	2,3	1,2

Under the matrix notation, the allocation process of mechanisms to assign the resource is matching the row and the column according to certain principles, deleting the row and the column once an allocation is finished, then continuing the match with new matrix.

Let's go back to the proof of theorem 1, under TTC and ASOM mechanisms, because of the consistent preference structure, there is no cycle exist, thus the process and outcome of these two mechanisms are same. The process in matrix could be described as follows: at first, consistency of preference structure is reflected by the left element in each column of matrix, which is either the same number or ∞ . Therefore, the allocation process is:

STEP 1: check out left values of elements in each column in matrix, choose the column in which the left value is 1(at least one value is 1, the others could be ∞), find out the element (value: (1,1)) whose right value is 1 in this column, then the row and the column where this element being is an outcome, it denotes that the resource of this column is assigned to the participant of this row. Then delete this row, and decrease one unit of this resource (if the resource number decrease to 0, delete this column), form a new matrix;

STEP 2：check out left values of elements in each column, choose the column in which the left value is 2(at least one value is 2, the others could be ∞), find out the element with the minimum value on right side in this column, then the row and the column where this element being is an outcome, it denotes that the resource of this column is assigned to the participant of this row. Then delete this row, and decrease one unit of this resource (if the resource number decrease to 0, delete this column), form a new matrix;

……

STEP k：check out left values of elements in each column, choose the column in which the left value is k(at least one value is k, the others could be ∞), find out the element with the minimum value on right side in this column, then the row and the column where this element being is an outcome, it denotes that the resource of this column is assigned to the participant of this row. Then delete this row, and decrease one unit of this resource (if the resource number decrease to 0, delete this column), form a new matrix;

If all resources or participants are matched, the allocation finishes. If the left value of all elements in a certain column is ∞ , then the allocation finish, including the participant in the row where this element is, although some of them haven't finished their allocations yet, all of them couldn't obtain any resource.

In the process above, we can see that, under the consistent preference condition, the outcome of TTC and ASOM mechanisms are decided by priority structure, because of different priority structures, right values of each column, which is the unique determinant of outcome, are not the same.

Under BOS mechanism, the allocation process is as follows:

STEP 1：check out left values of elements in each column, （numerical value equal in the same column of some is ∞ ）, in accordance with the left value of the order from small to large, find out the element with right value being 1 in each column, it denotes that the resource of this column is assigned to the participant of this row. Then delete this row, and decrease one unit of this resource (if the resource number decrease to 0, delete this column), keep on repeating the above procedure, until there is no element exist with right value being 1 in each column;

……;

STEP k：check out left values of elements in each column, （numerical value equal in the same column of some is ∞ ）, in accordance with the left value of the order from small to large, find out the element with right value being k in each column, it denotes that the resource of this column is assigned to the participant of this row. Then delete this row, and decrease one unit of this resource(if the resource number decrease to 0, delete this column), keep on repeating the above procedure, until there is no element exist with right value being k in each column;

When all resources or participants are matched, the allocation finishes.

From the allocation producer of BOS mechanism, we could see that, at first, allocation is from every resource priority rankings list, searching for the participant who has relatively top priority, especially in the case of inconsistent high degree of priority(in different resources, the participant has different priority rankings), BOS mechanism become into a simple process of searching for participant who has relatively top priority, different priority structures must lead to different outcomes. Therefore, in the condition of consistent preference, the outcome of BOS mechanism is also decided by priority structure.

Proof of theorem 2

At first, under TTC and ASOM mechanism, if the outcome is unfair, there exist participants i and j, obtain resource x and y respectively, but yp_ix while $f_y(i) < f_y(j)$, thus, from the allocation producer above, we can see that, resource y reaches resource allocation prior to resource x, and in the process to distribute resource y, i couldn't get y, therefore, the right value of elements in row i column y is bigger than row j column y's, in another word, for resource y, the priority of participant i is lower than j's, contradiction! Therefore, the allocation outcome is fair. Actually, ASOM originally is a fair mechanism, while under the condition of consistent preference, TTC and ASOM mechanisms are equivalent, thus, that their outcomes are fair, is very reasonable.

As for consistency, in the condition of consistent preference, allocation producer works orderly according to all participants' consistent preference orders for resource. Firstly, allocate the resource on top preference rankings, and then the resource on the back, while the allocation of certain resource is completely decided by priority ranking, the back one obtaining the resource is always later than front ones. From the matrix notation, once finish a participant's allocation, the corresponding row would be deleted, and supply of resource would be decreased 1 unit, farther more, once this kind of resource is assigned out, this column which represents the resource would be deleted, and a new matrix is formed, the process goes on. Obviously, the allocation problem which new matrix represent, is a sub-problem of original one, and the solution process of original one include solutions of a lot of sub-problems. Therefore, this kind of matrix allocation process which is equal to the mechanism; actually implies that the outcome must be consistent.

Obviously, TTC and AOSM mechanism are efficient, because, TTC mechanism originally is efficient, and AOSM mechanism reaches participant Pareto-efficiency, therefore, it is also efficient.

Second, under BOS mechanism, preference consistency could not grantee its fairness, for an example:

	a	B	y	x
i	1,2	2,4	3,2	4,1
j	1,3	2,3	3,3	4,4
k	1,1	2,2	3,1	4,2
l	1,4	2,1	3,3	4,3

Under BOS mechanism, the outcome is $\begin{Bmatrix} a & b & x & y \\ k & l & i & j \end{Bmatrix}$, but yp_ix and $f_y(i) < f_y(j)$, thus, it is not fair.

As for consistency, according to matrix allocation producer, once a participant finish his/her allocation, delete the row the participant represents and corresponding resource number decrease 1 unit, then form a new matrix. And the new matrix reflects a sub-problem of original problem, because the preference and priority ranking of remained participants are not changed, the result of new matrix in accordance with allocation program is as same as taking it as an independent problem, therefore, in the condition of consistent preference structure, outcomes of BOS mechanism are consistent.

As for efficiency, if the outcome under mechanism μ of BOS is not efficient, there exists another mechanism y μ', such that some participants' utilities increase and there is no change on others'. Here are these two situations, one is with consistent outcomes of others', one participant i obtain an allocation a under mechanism μ' and noting under mechanism μ, however, it wouldn't happen in BOS matrix allocation producer, because the others' allocations are consistent, it indicate that there exists surplus of resource a under mechanism μ, and the second reason is i's utility increases under mechanism μ', it implies $ap_i\phi$, in BOS mechanism, surplus of resource a denotes that the column where a is hasn't been deleted, and so does i. $ap_i\phi$ indicates that under BOS mechanism, i could obtain resource a; the other situation is with consistent outcomes of other participants, there exist two participants i and j, their utilities both increase after they change the allocation with each other, let $\mu(i)=a$, $\mu(j)=b$, $\mu'(i)=b$, $\mu'(j)=a$, then bp_ia, ap_jb, which is contrary to the hypothesis of consistent preference structure, thus this would never happen, neither, therefore, in the condition of consistent preference structure, BOS mechanism is efficient.

References

Gladly provided upon request.

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Xinyu PENG received the Ph.D from Sun Yat-sen University , now is the associate professor of School of Business Administration, South China University of Technology.

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